Partial Derivatives

Wednesday, May 31, 2023 9:24 AM

more than one

derivatives for $f(x,y)$:

$\frac{\partial}{\partial x} f(x,y)$

- partial derivative for x: $\partial_x f(x,y)$ measures variation in direction of x-axis
- partial derivative for y: $\partial_y f(x,y)$ measures variation in direction of y-axis

how to compute $\partial_x f(x,y)$:

1) think of y as being "constant"
2) derive as usual with respect to x

→ one variable derivative

same for $\partial_y f(x,y)$ just now thinking of x as constant

ex 1) $f(x,y) = x^2 + y^2$

- $\partial_x f(x,y) = 2x$
- $\partial_y f(x,y) = 2y$

ex 2) $f(x,y) = x \cdot y$

- $\partial_x f(x,y) = y$
- $\partial_y f(x,y) = x$

ex 3) $f(x,y) = x^2 \cdot y$

- $\partial_x f(x,y) = y \cdot 2x$
- $\partial_y f(x,y) = x^2$

ex 4) $f(x,y) = e^{xy^2} + xy\cos(x)$

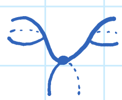
- $\partial_x f(x,y) = e^{xy^2} \cdot y^2 + y(\cos(x) - \sin(x) \cdot x)$
- $\partial_y f(x,y) = e^{xy^2} \cdot 2xy + x\cos(x)$

x = constant, so cos (x) = constant

* chain rule still applies *

application: computing critical points


min


saddle


max

1) compute $\partial_x f$ & $\partial_y f$
2) solve $\partial_x f = 0$ & $\partial_y f = 0$

* similar to using $f'(x) = 0$ to solve for max or min *

ex 1) $f(x,y) = x^2 - y^2$

- $\partial_x f = 2x$ $\qquad 2x = 0 \longrightarrow x = 0$
- $\partial_y f = -2y$ $\qquad -2y = 0 \longrightarrow y = 0$

$\boxed{(x,y) = (0,0) = \text{critical point}}$